**Solutions:**

1. See attached derivation of conditional variance formula
2. Section 5.1.4; much easier analysis method using Foster-Lyapunov theorem (see Srikant’s book, Theorem 3.3.7, page 62), with Lyapunov function . From and the expansion on slide 9:

The required negative drift is guaranteed for large enough provided that . This proves that is a sufficient condition.

Necessity is established next. The case clearly leads to a contradiction (from *G/G/1* analysis), so we need to show that for the MC is not positive recurrent; see page 235 in reference book.